International Journal of Theoretical Physics, Vol. 41, No. 5, May 2002 (© 2002)

Čerenkov Effect With Massive Photons

Miroslav Pardy[1]

Received August 13, 2001

The equations of massive electrodynamics are derived and the power spectrum formula for the Čerenkov radiation of massive photons is found. It is argued that the massive Čerenkov effect can be observed in superconductive media, ionosphere plasma, waveguides, and in particle laboratories.

KEY WORDS: Čerenkov effect; source theory; photons.

1. INTRODUCTION

The possibility that photon may be a massive particle has been treated by many physicists. At the present time great attention is devoted to discussion of the mass of the neutrino and its oscillations; nevertheless, the theoretical problems with massive photons are of the same importance. The established fact is that massive electrodynamics is a perfectly consistent classical and quantum field theory (Feldman and Mathews, 1963; Goldhaber and Nieto, 1971; Minkowski and Seiler, 1971). In all respect the quantum version has the same status as the standard QED with massless photons. In this paper we do not solve the radiative problems in sense of Nieuwenhuizen (1973); our goal is to determine the Čerenkov effect of massive photons which is not analyzed in that paper.

In particle physics and quantum field theory (Commins and Bucksbaum 1983; Ryder, 1985; de Wit and Smith, 1986) the photon is defined as a massless particle with spin 1. Its spin is along or in opposite direction to its motion. The massive photon as a neutral massive particle is usually called vector boson. There are other well-known examples of massive spin 1 particles, for instance, neutral ϱ-meson, φ-meson, J/ψ particle, and bosons $W^{\pm}$ and Z^0 in particle physics.

While the massless photon is described by the Maxwell lagrangian, the massive photon is described by the Proca lagrangian, from which the field equations follow. Massive electrodynamics can be considered as a generalization of massless

[1] Department of Theoretical Physics and Astrophysics, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic; e-mail: pamir@physics.muni.cz.

electrodynamics. Well-known areas where the massive photon or boson play a substantial role are superconductivity (Ryder, 1985), plasma physics (Anderson, 1963), waveguides, and so on. So, the physics of the massive photon is meaningful and this means that the Čerenkov effect with massive photons is also worth investigating.

In order to be pedagogically clear, in Section 2 we treat the massive spin 0 quantum field theory and then in Section 3 the massive spin 1 field theory. In Section 4 the massive Maxwell equations are derived. In that section a power spectral formula is derived for the massive Čerenkov radiation for the case of one charge moving in a medium.

2. MASSIVE SPIN 0 FIELDS

We begin with the massive spin 0 field as the most simple illustration of how source theory works (Schwinger, 1970). The action of spin 0 particles is according to source theory composed from the scalar source $K(x)$ and propagator Δ_+ in such a way that it gives the correct probability condition for the vacuum to vacuum amplitude. We show here that the action is

$$W(K) = \frac{1}{2}\int (dx)(dx')K(x)\Delta_+(x-x')K(x'), \tag{1}$$

and gives the right probability condition $|\langle 0_+ \mid 0_-\rangle|^2 \leq 1$ (Schwinger, 1970; Schwinger *et al.*, 1976), where ($\hbar = 1$)

$$\langle 0_+ \mid 0_-\rangle^K = e^{iW(K)} \tag{2}$$

is the basic formula of the Schwinger source theory, with $\langle 0_+ \mid 0_-\rangle$ being the vacuum to vacuum amplitude.

In order to prove that the quantity $\langle 0_+ \mid 0_-\rangle$ is really the vacuum to vacuum amplitude it is necessary to know the explicit form of the Green function $\Delta_+(x-x')$ which satisfies the equation

$$(-\partial^2 + m^2)\Delta_+(x-x') = \delta(x-x'). \tag{3}$$

From the last equation it follows that

$$\Delta_+(x-x') = \frac{1}{(-\partial^2+m^2)}\int \frac{(dp)}{(2\pi)^4}e^{ip(x-x')}. \tag{4}$$

The formula (4) is not unambiguous and it is necessary to specify it by the ε-term, or

$$\Delta_+(x-x') = \int \frac{(dp)}{(2\pi)^4}\frac{e^{ip(x-x')}}{p^2+m^2-i\varepsilon};\quad \varepsilon \to 0_+. \tag{5}$$

Now, let us prove that $|\langle 0_+ \mid 0_- \rangle|^2$ is the probability of the persistence of vacuum. According to definition

$$\langle 0_+ \mid 0_- \rangle = \exp\left\{ \frac{i}{2} \int (dx)(dx') K(x) \Delta_+(x - x') K(x') \right\}, \tag{6}$$

we have

$$\begin{aligned} \langle 0_+ \mid 0_- \rangle &= \exp\left\{ \frac{i}{2} \int (dx)(dx') \int \frac{(dp)}{(2\pi)^4} K(x) \frac{e^{ip(x-x')}}{p^2 + m^2 - i\varepsilon} K(x') \right\} \\ &= \exp\left\{ \frac{i}{2} \int \frac{(dp)}{(2\pi)^4} \frac{K(p)K(-p)}{p^2 + m^2 - i\varepsilon} \right\} \\ &= \exp\left\{ \frac{i}{2} \int \frac{(dp)}{(2\pi)^4} \frac{|K(p)|^2}{p^2 + m^2 - i\varepsilon} \right\} \end{aligned} \tag{7}$$

as a consequence of Eq. (5) and $K^*(p) = K(-p)$. Using the well-known theorem

$$\frac{1}{x - i\varepsilon} = P\left(\frac{1}{x}\right) + i\pi\delta(x); \quad \varepsilon \to 0, \tag{8}$$

where P denotes the principal value of integral, we get the following formula for the vacuum persistence:

$$|\langle 0_+ \mid 0_- \rangle|^2 = e^{-2\,\mathrm{Im}\,W} = \exp\left\{ -2 \int \frac{(dp)}{(2\pi)^4} \pi |K(p)|^2 \delta(p^2 + m^2) \right\}. \tag{9}$$

Using

$$\begin{aligned} \delta(p^2 + m^2) = \frac{1}{2(\mathbf{p}^2 + m^2)^{1/2}} &\left\{ \delta\left(p^0 - (\mathbf{p}^2 + m^2)^{1/2}\right) \right. \\ &\left. + \delta\left(p^0 + (\mathbf{p}^2 + m^2)^{1/2}\right) \right\}, \end{aligned} \tag{10}$$

we get

$$2 \int \frac{(dp)}{(2\pi)^4} \pi |K(p)|^2 \delta(p^2 + m^2) = \int \frac{(d\mathbf{p})}{(2\pi)^3} \frac{1}{2p^0} |K(p^0, \mathbf{p})|^2, \tag{11}$$

and then,

$$|\langle 0_+ \mid 0_- \rangle|^2 = \exp\left\{ - \int d\omega_p |K(p)|^2 \right\}, \tag{12}$$

where

$$d\omega_p = \frac{(d\mathbf{p})}{(2\pi)^3} \frac{1}{2p^0} \quad p^0 = +(\mathbf{p}^2 + m^2)^{1/2}. \tag{13}$$

The expression (12) shows that in the presence of the scalar source $K(x)$ the probability for vacuum to remain a vacuum is equal to or less than 1.

Now, let us show the derivation of the field equation from the action W for the scalar field φ, where

$$\begin{aligned} W &= \frac{1}{2}\int K\Delta_+ K = \frac{1}{2}\int \varphi K = \frac{1}{2}\int \varphi(-\partial^2 + m^2)\varphi \\ &= \frac{1}{2}\int (\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2) = 2W - W \\ &= \int \varphi K - \frac{1}{2}[(\partial\varphi)^2 + m^2\varphi^2] = \int (dx)[K(x)\varphi(x) + \mathcal{L}(\varphi(x))] \end{aligned} \tag{14}$$

with

$$\mathcal{L}(\varphi(x)) = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2]. \tag{15}$$

Let us put

$$\delta_\varphi W = 0, \tag{16}$$

or

$$\int \delta\varphi K - [\partial_\mu\varphi\partial^\mu\delta\varphi + m^2\varphi\delta\varphi] = 0. \tag{17}$$

After some modification we get

$$\int (dx)[K - (-\partial_\mu\partial^\mu\varphi + m^2\varphi)]\delta\varphi = 0. \tag{18}$$

As variable φ is an arbitrary one the last integral is equal to zero only if

$$(-\partial^2 + m^2)\varphi(x) = K(x), \tag{19}$$

which is the Klein–Gordon equation with source $K(x)$ on the right side of equation. Now, let us derive the Proca equation for massive particles with spin 1 and generate the Maxwell equations for massive photons.

3. MASSIVE SPIN 1 FIELDS

We show the natural construction of the field of the particles with spin 1. The derivation of the action for this massive spin 1 fields is based on the modification of the derivation of spin 0 fields.

The relation

$$|\langle 0_+ \mid 0_-\rangle^2 = \exp\{-2\,\mathrm{Im}\,W\} \leq 1 \tag{20}$$

is postulated to be valid for all spin fields. Let us show here the construction of action and field equations concerning spin 1.

If spin 0 particles and fields are described by the scalar source, then a vector source denoted here as $J^{\mu}(x)$ can be considered as a candidate for the description of the spin 1 fields and particles. However, there exist some obstacles because source $J^{\mu}(x)$ has four components and spin 1 particles have only three spin possibilities. Nevertheless, first let us investigate by analogy with the spin 0 fields the following form of the action for the unit spin fields:

$$W(J) = \frac{1}{2}\int (dx)(dx')J^{\mu}(x)\Delta_{+}(x-x')J_{\mu}(x'). \tag{21}$$

Then,

$$|\langle 0_{+} \mid 0_{-}\rangle|^2 = e^{iW}\, e^{-iW^{*}} = \exp\left\{-\int d\omega_p\, J^{*\mu}(p)J_{\mu}(p)\right\}. \tag{22}$$

However,

$$J^{*\mu}(p)J_{\mu}(p) = |\mathbf{J}(p)|^2 - |J^0(p)|^2 \leq 0 \;\text{ or }\; > 0, \tag{23}$$

and it means that the quantity defined by Eq. (21) cannot be considered as the probability of the persistence of vacuum.

The difficulty can be overcome by replacing the original form $J^{*\mu}(x)J_{\mu}(x)$ by the following invariant structure:

$$J^{*\mu}(p)\left[g_{\mu\nu} + \frac{1}{m^2}p_{\mu}p_{\nu}\right]J^{\nu}(p), \tag{24}$$

which can be, with regard to its invariancy, determined in the rest frame of the timelike vector p^{μ}, where $p^{\mu} = (m, 0, 0, 0)$ in the rest frame. Then, with $g_{\alpha\alpha} = (-1, 1, 1, 1)$ and $g_{\mu\nu} = 0$ for $\mu \neq \nu$, we have

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2}p_{\mu}p_{\nu} = \begin{cases} \delta_{kl}; & \mu = k;\ \nu = l \\ 0; & \mu = 0;\ \nu = 0 \\ 0; & \mu = k;\ \nu = 0 \end{cases} \tag{25}$$

and

$$J^{*\mu}(p)\bar{g}_{\mu\nu}J^{\nu}(p) \equiv |\mathbf{J}|^2, \tag{26}$$

and now the quantity $|\langle 0_{+} \mid 0_{-}\rangle|^2$ can be interpreted as the vacuum persistence probability.

At the same time $|\mathbf{J}|^2$ contains three independent source components, transforming among themselves under spatial rotation, as it is appropriate to unit spin.

After using Eq. (24) it may be easy to get $W(J)$ in the space-time representation by the Fourier transformation, as follows:

$$W(J) = \frac{1}{2}\int (dx)(dx')\left\{J_{\mu}(x)\Delta_{+}(x-x')J^{\mu}(x') + \frac{1}{m^2}\partial_{\mu}J^{\mu}(x)\Delta_{+}(x-x')\partial'_{\nu}J^{\nu}(x')\right\}. \tag{27}$$

The field of spin 1 particles can be defined using the definition of the test source $\delta J^{\mu}(x)$ by the relation

$$\delta W(J) = \int (dx)\delta J^{\mu}(x)\varphi_{\mu}(x), \tag{28}$$

where φ_{μ} is the field of particles with spin 1. After performing variation of the formula (27) and comparison with Eq. (28) we get the equation for field of spin 1 in the following form:

$$\varphi_{\mu}(x) = \int (dx')\Delta_{+}(x-x')J_{\mu}(x') - \frac{1}{m^2}\partial_{\mu}\int (dx')\Delta_{+}(x-x')\partial'_{\nu}J^{\nu}(x'). \tag{29}$$

The divergence of the vector field $\varphi_{\mu}(x)$ is given by the relation

$$\begin{aligned}\partial_{\mu}\varphi^{\mu}(x) &= \int (dx')\Delta_{+}(x-x')\partial'_{\mu}J^{\mu}(x') - \frac{1}{m^2}\partial^2\int (dx')\Delta_{+}(x-x')\partial'_{\nu}J^{\nu}(x') \\ &= \frac{1}{m^2}\partial_{\mu}J^{\mu}(x),\end{aligned} \tag{30}$$

as a consequence of Eq. (3) or

$$-\partial^2\Delta_{+} = \delta(x-x') - m^2\Delta_{+}. \tag{31}$$

Further, after applying operator $(-\partial^2 + m^2)$ on Eq. (29) we have the following equations:

$$(-\partial^2 + m^2)\varphi_{\mu}(x) = J_{\mu}(x) - \frac{1}{m^2}\partial_{\mu}\partial_{\nu}J^{\nu}(x), \tag{32}$$

$$(-\partial^2 + m^2)\varphi_{\mu}(x) + \partial_{\mu}\partial_{\nu}\varphi^{\nu}(x) = J_{\mu}(x), \tag{33}$$

as a consequence of Eq. (30).

It may be easy to cast the last equation into the following form:

$$\partial^{\nu}G_{\mu\nu} + m^2\varphi_{\mu} = J_{\mu}, \tag{34}$$

where

$$G_{\mu\nu}(x) = -G_{\nu\mu}(x) = \partial_{\mu}\varphi_{\nu} - \partial_{\nu}\varphi_{\mu}. \tag{35}$$

Identifying $G_{\mu\nu}$ with $F_{\mu\nu}$ of the electromagnetic field we get instead of Eqs. (33) and (34) the so-called Proca equation for the electromagnetic field with the massive photon:

$$(-\partial^2 + m^2)A_{\mu}(x) + \partial_{\mu}\partial_{\nu}A^{\nu}(x) = J_{\mu}(x), \tag{36}$$

$$\partial^{\nu}F_{\mu\nu} + m^2A_{\mu} = J_{\mu}, \tag{37}$$

$$F_{\mu\nu}(x) = -F_{\nu\mu}(x) = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{38}$$

In case $m^2 \neq 0$, we can put $\partial_\mu A^\mu = 0$ to get

$$(-\partial^2 + m^2)A_\mu(x) = 0, \qquad \partial_\mu A^\mu = 0. \tag{39}$$

The solution of the system (39) is the plane wave

$$A_\mu = \varepsilon_\mu(\mathbf{k})e^{ikx}, \qquad k^2 = -m^2, \tag{40}$$

with $k\varepsilon(\mathbf{k}) = 0$, which is precisely the correct definition of a massive particle with spin 1. We will see in the next section how to generalize this procedure to the situation of the massive electrodynamics in dielectric and magnetic media and then to apply it in the determination of the massive Čerenkov radiation.

Equation (34) can also be derived from the action

$$W = \int (dx)(J^\mu(x)\varphi_\mu(x) + \mathcal{L}(\varphi(x))), \tag{41}$$

where

$$\mathcal{L} = -\frac{1}{2}\left(\frac{1}{2}(\partial^\mu\varphi^\nu - \partial^\nu\varphi^\mu)(\partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu) + m^2\varphi^\mu\varphi_\mu\right), \tag{42}$$

where we have used the arrangement

$$\int (dx)\varphi^\mu(-\partial^2)\varphi_\mu = \int (dx)\partial^\nu\varphi^\mu\partial_\nu\varphi_\mu \tag{43}$$

and

$$\int (dx)\varphi^\mu\partial_\mu\partial^\nu\varphi_\nu = -\int (dx)\varphi^\nu\partial^\mu\partial_\mu\varphi_\nu = -\int (dx)\varphi_\mu\partial^\mu\partial^\nu\varphi_\nu. \tag{44}$$

Using the last equation (44) we get the lagrange function in the following standard form:

$$\mathcal{L} = -\frac{1}{2}(\partial^\nu\varphi^\mu\partial_\nu\varphi_\mu - (\partial_\mu\varphi^\mu)^2 + m^2\varphi^\mu\varphi_\mu). \tag{45}$$

If we use the A- and F-symbols, we receive from Eq. (42) the Proca lagrangian

$$\mathcal{L} = -\frac{1}{2}\left(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2A^\mu A_\mu\right), \tag{46}$$

or

$$\mathcal{L} = -\frac{1}{2}(\partial^\nu A^\mu\partial_\nu A_\mu - (\partial_\mu A^\mu)^2 + m^2A^\mu A_\mu). \tag{47}$$

By variation of the corresponding lagrangians for the massive field with spin 1, we get evidently the massive Maxwell equations.

It is evident that the zero mass limit does not exist for $\partial_\mu J^\mu(x) \neq 0$. Thus, we are forced to redefine action $W(J)$. One of the possibilities is to put

$$\partial_\mu J^\mu(x) = mK(x) \tag{48}$$

and identify $K(x)$ in the limit $m \to 0$ with the source of massless spin 0 particles. Since the zero mass particles with zero spin are experimentally unknown in any event, we take $K(x) = 0$ and we write

$$W_{[m=0]}(J) = \frac{1}{2}\int (dx)(dx')J_\mu(x)D_+(x-x')J^\mu(x'), \tag{49}$$

where

$$\partial_\mu J^\mu(x) = 0 \tag{50}$$

and

$$D_+(x-x') = \Delta_+(x-x'; m=0). \tag{51}$$

In case we want to work with electrodynamics in a medium it is necessary to involve such parameters as velocity of light, c, magnetic permeability μ, and the dielectric constant ϵ. Then the corresponding equations for electromagnetic potentials which are compatible with the Maxwell equations are as follows (Schwinger *et al.*, 1976):

$$\left(\Delta - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)J_\nu, \tag{52}$$

where the corresponding Lorentz gauge is defined in Schwinger *et al.* (1976) in the following form:

$$\partial_\mu A^\mu - (\mu\epsilon - 1)(\eta\partial)(\eta A) = 0, \tag{53}$$

where $\eta^\mu = (1, \mathbf{0})$ is the unit timelike vector in the rest frame of the medium. The four potentials are $A^\mu = (\phi, \mathbf{A})$ and the four currents are $J^\mu = (c\varrho, \mathbf{J})$; n is the index of refraction of this medium.

The corresponding Green function $D_+^{\mu\nu}$ in the x-representation is

$$D_+^{\mu\nu}(x-x') = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)D_+(x-x'). \tag{54}$$

$D_+(x-x')$ was derived by Schwinger *et al.* (1976) as follows:

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4}\frac{e^{ik(x-x')}}{|\mathbf{k}^2| - n^2(k^0)^2 - i\varepsilon}, \tag{55}$$

or

$$D_+(x-x') = \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega\,\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{56}$$

4. MASSIVE PHOTON IN ELECTRODYNAMICS AND THE ČERENKOV EFFECT

The massive electrodynamics in a medium can be constructed by generalization of massless electrodynamics to the case with massive photon. In our case it means that we replace only Eq. (52) by the following one:

$$\left(\Delta - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2} + \frac{m^2c^2}{\hbar^2}\right) A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right) J_\nu, \qquad (57)$$

where m is mass of photon. The Lorentz gauge (53) is also conserved in the massive situation.

In superconductivity, photon is a massive spin 1 particle as a consequence of a broken symmetry of the Landau–Ginzburg lagrangian. The Meissner effect can be used as a experimental demonstration that photon in a superconductor is a massive particle. In particle physics the situation is analogous to the situation in superconductivity. The masses of particles are also generated by the broken symmetry or, in other words, by the Higgs mechanism. Massive particles with spin 1 form the analogue of the massive photon.

Kirzhnitz and Linde proposed a qualitative analysis wherein they indicated that, as in the Ginzburg–Landau theory of superconductivity, the Meissner effect can also be realized in the Weinberg model. Later, it was shown that the Meissner effect is realizable in renormalizable gauge fields and also in the Weinberg model (Kirzhnitz and Linde 1972; Yildiz, 1977).

We concentrate in this paper on the Čerenkov radiation with massive photons. The so-called Čerenkov radiation was observed experimentally first by Čerenkov (1934) and explained theoretically by Tamm and Frank (1937) in classical electrodynamics as a shock wave resulting from a charged particle moving through a material faster than the velocity of light in the material. The source theory explanation was given by Schwinger *et al.* (1976) and the particle production by the Čerenkov mechanism was discussed by Pardy (1983a,b). The Čerenkov effect at finite temperature in source theory was discussed by Pardy (1989, 1995) and the Čerenkov effect with radiative corrections in electromagnetism and gravity was analyzed by Pardy (1994a,b).

We will investigate how the spectrum of the Čerenkov radiation is modified if we suppose that the massive photons are generated instead of massless photons. The derived results form an analogue of the situation with massless photons. According to Pardy (1989; 1994a,b; 1995) and Dittrich (1978) with the analogy of the massless photon propagator $D(k)$ in the momentum representation

$$D(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\varepsilon}, \qquad (58)$$

the massive photon propagator is of the form (here we introduce $\hbar$ and c)

$$D(k, m^2) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 + \frac{m^2c^2}{\hbar^2} - i\varepsilon}, \tag{59}$$

where this propagator is derived from an assumption that the photon energetical equation is

$$|\mathbf{k}|^2 - n^2(k^0)^2 = -\frac{m^2c^2}{\hbar^2}, \tag{60}$$

where n is the parameter of the medium and m is mass of photon in this medium.

From Eq. (60) the dispersion law for the massive photons follows:

$$\omega = \frac{c}{n}\sqrt{k^2 + \frac{m^2c^2}{\hbar^2}}. \tag{61}$$

Let us remark here that such dispersion law is valid not only for the massive photon but also for electromagnetic field in waveguides and electromagnetic field in ionosphere. It means that the corresponding photons are also massive and the theory of massive photons is physically meaningful. This means that the Čerenkov radiation of massive photons is also physically meaningful and it is worthwhile to study it.

The validity of Eq. (60) can be verified using very simple idea that for $n = 1$ the Einstein equation for mass and energy has to follow. Putting $\mathbf{p} = \hbar\mathbf{k}$, $\hbar k^0 = \hbar(\omega/c) = E/c$, we get the Einstein energetical equation

$$E^2 = \mathbf{p}^2c^2 + m^2c^4. \tag{62}$$

The propagator for the massive photon is then derived as

$$D_+(x - x', m^2) = \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega \frac{\sin\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{63}$$

The function (63) differs from the the original function D_+ by the factor

$$\left(\frac{\omega^2n^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right)^{1/2}. \tag{64}$$

From Eqs. (56) and (63) the potentials generated by the massless and massive photons respectively follow. In case of the massless photon, the potential is according to Schwinger defined by the formula:

$$\begin{aligned} V(\mathbf{x} - \mathbf{x}') &= \int_{-\infty}^{\infty} d\tau\, D_+(\mathbf{x} - \mathbf{x}', \tau) \\ &= \int_{-\infty}^{\infty} d\tau \left\{\frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|\tau|}\right\}. \end{aligned} \tag{65}$$

The τ-integral can be evaluated using the mathematical formula

$$\int_{-\infty}^{\infty} d\tau\, e^{-i\omega|\tau|} = \frac{2}{i\omega} \tag{66}$$

and the ω-integral can be evaluated using the formula

$$\int_{0}^{\infty} \frac{\sin ax}{x}\, dx = \frac{\pi}{2}, \quad \text{for } a > 0. \tag{67}$$

After using Eqs. (66) and (67), we get

$$V(\mathbf{x} - \mathbf{x}') = \frac{1}{c}\frac{1}{4\pi}\frac{1}{|\mathbf{x} - \mathbf{x}'|}. \tag{68}$$

In case of the massive photon, the mathematical determination of potential is analogical to the massless situation with the only difference being that we use the propagator (63) and the table integral (Gradshteyn and Ryzhik, 1965):

$$\int_{0}^{\infty} \frac{dx}{x} \sin\left(p\sqrt{x^2 - u^2}\right) = \frac{\pi}{2} 2^{-pu}. \tag{69}$$

Using this integral we get that the potential generated by the massive photons is

$$V(\mathbf{x} - \mathbf{x}', m^2) = \frac{1}{c}\frac{1}{4\pi}\frac{\exp\left\{-\frac{mcn}{\hbar}|\mathbf{x} - \mathbf{x}'|\right\}}{|\mathbf{x} - \mathbf{x}'|}. \tag{70}$$

If we compare the potentials concerning massive and massless photons, we can deduce that Čerenkov radiation with massive photons can also be generated. So, the determination of the Čerenkov effect with massive photons is physically meaningful.

In case of the massive electromagnetic field in the medium, the action W is given by the following formula:

$$W = \frac{1}{2c^2} \int (dx)(dx') J^{\mu}(x) D_{+\mu\nu}(x - x', m^2) J^{\nu}(x'), \tag{71}$$

where

$$D_{+}^{\mu\nu} = \frac{\mu}{c}[g^{\mu\nu} + (1 - n^{-2})\eta^{\mu}\eta^{\nu}] D_{+}(x - x', m^2), \tag{72}$$

where $\eta^{\mu} \equiv (1, \mathbf{0})$, $J^{\mu} \equiv (c\varrho, \mathbf{J})$ is the conserved current, μ is the magnetic permeability of the medium, ϵ is the dielectric constant of the medium, and $n = \sqrt{\epsilon\mu}$ is the index of refraction of the medium.

The probability of the persistence of vacuum follows from the vacuum amplitude (2) in the following form:

$$|\langle 0_{+} \mid 0_{-}\rangle|^2 = e^{-\frac{2}{\hbar}\operatorname{Im} W}, \tag{73}$$

where Im W is the basis for the definition of the spectral function $P(\omega, t)$:

$$-\frac{2}{\hbar}\mathrm{Im}\, W \stackrel{d}{=} -\int dt\, d\omega \frac{P(\omega, t)}{\hbar\omega}. \tag{74}$$

Now, if we insert Eq. (72) into Eq. (71), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}\, d\mathbf{x}'\, dt' \left[\frac{\sin\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|}\right]$$
$$\times \cos[\omega(t - t')][\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t)\cdot\mathbf{J}(\mathbf{x}', t')]. \tag{75}$$

Now, let us apply the formula (75) in order to get the Čerenkov distribution of massive photons. The Čerenkov radiation is produced by charged particle of charge Q moving at a constant velocity $\mathbf{v}$. Thus, we can write the charge density and the current density as

$$\varrho = Q\delta(\mathbf{x} - \mathbf{v}t), \qquad \mathbf{J} = Q\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{76}$$

After insertion of Eq. (76) into Eq. (75), we get ($v = |\mathbf{v}|$)

$$P(\omega, t) = \frac{Q^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)$$
$$\times \int_{\infty}^{\infty}\frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2} v\tau\right)\cos\omega\tau, \tag{77}$$

where we have put $\tau = t' - t$, $\beta = v/c$.

For $P(\omega, t)$, the situation leads to evaluation of the τ-integral. For this integral we have

$$\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2} - \frac{c^2}{m^2}\right]^{1/2} v\tau\right)\cos\omega\tau = \begin{cases}\pi, & 0 < m^2 < \frac{\omega^2}{c^2v^2}(n^2\beta^2 - 1)\\ 0, & m^2 > \frac{w^2}{c^2v^2}(n^2\beta^2 - 1).\end{cases} \tag{78}$$

From Eq. (78) it immediately follows that $m^2 > 0$ implies that the Čerenkov threshold $n\beta > 1$. From Eqs. (77) and (78) we get the spectral formula of the Čerenkov radiation of massive photons in the form

$$P(\omega, t) = \frac{Q^2}{4\pi}\frac{v\omega\mu}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right) \tag{79}$$

for

$$\omega > \frac{mcv}{\hbar}\frac{1}{\sqrt{n^2\beta^2 - 1}} > 0, \tag{80}$$

and $P(\omega, t) = 0$ for

$$\omega < \frac{mcv}{\hbar} \frac{1}{\sqrt{n^2\beta^2 - 1}}. \tag{81}$$

Using the dispersion law (61) we can write the power spectrum $P(\omega)$ as a function dependent on k^2. Then,

$$P(k^2) = \frac{Q^2}{4\pi} \frac{v\mu}{nc} \sqrt{k^2 + \frac{m^2c^2}{\hbar^2}} \left(1 - \frac{1}{n^2\beta^2}\right), \quad k^2 > \frac{m^2c^2}{\hbar^2} \frac{1}{n^2\beta^2 - 1} \tag{82}$$

and $P(\omega, t) = 0$ for $k^2 < (m^2c^2/\hbar^2)(n^2\beta^2 - 1)^{-1}$.

The most simple way of how to get the angle Θ between vectors $\mathbf{k}$ and $\mathbf{p}$ is the use of the conservation laws for energy and momentum.

$$E - \hbar\omega = E', \tag{83}$$

$$\mathbf{p} - \hbar\mathbf{k} = \mathbf{p}', \tag{84}$$

where E and E' are energies of a moving particle before and after the emission of a photon with energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$, and $\mathbf{p}$ and $\mathbf{p}'$ are momenta of the particle before and after the emission of the same photon.

If we raise Eqs. (83) and (84) to the second power and take the difference of these quadratic equations, we can extract the $\cos\Theta$ in the form

$$\cos\Theta = \frac{1}{n\beta}\left(1 + \frac{m^2c^2}{\hbar^2k^2}\right)^{1/2} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right) - \frac{m^2c^2}{2n^2p\hbar k}, \tag{85}$$

which has the correct massless limit. The massless limit also gives the sense of the parameter n which is introduced in the massive situation. We also observe that while in the massless situation the angle of emission depends only on $n\beta$, and in case of massive situation it also depends on the wave vector k. It means that the emission of the massive photons are emitted by the Čerenkov mechanism in all space directions. So, in the experiment the Čerenkov production of massive photons can be strictly distinguished from the Čerenkov production of massless photons or from the hard production of spin 1 massive particles.

5. DISCUSSION

The distribution of massive photons generated by the Čerenkov radiation is derived here, to our knowledge, in the framework of the source theory for the first time and there is no conventional derivation of this effect in QED. As this effect was not discussed in physical literature, we fill up the gap by this paper.

The velocity of the charged projectile which generates the massless Čerenkov radiation can be considered constant during the process of radiation because the energy loss due to radiative process is small. However, in case of massive Čerenkov effect the energy loss of the projectile may be large, which means the projectile is strongly deccelerated. It means the duration of the generation of massive photons is very short. The velocity can be considered constant only in the case of a very energetical and heavy charged projectile.

From the theoretical point of view, we used the massive electrodynamics which is only the generalization of the massless electrodynamics. So, our derivation of the Čerenkov radiation of massive photons can also be considered as a generalization of the situation with the massless photons.

The theory of the Čerenkov radiation of massive photons concerns the photons not only in superconductive medium but also in plasma medium in electron gas and ionosphere medium or photons in waveguides. The possibility of the existence of the massive photons in neutron stars is discussed by Voskresensky *et al.* (1998). The bosons $W^{\pm}$ and Z^0 are also massive and it means that the generalization of our approach to the situation in the standard model is evidently feasible. Similarly, the generation of vector mesons, ρ, φ, J/ψ by the Čerenkov mechanism may be possible. Probably, they can be generated in a such nuclear medium where they play role of mediators of nuclear forces.

In the experiment the Čerenkov effect with massive photons can be strongly distinguished from the classical effect because the emission of massive photons is distributed in all space directions.

We hope that with regard to the situation in physics of superconductivity, plasma physics, physics of ionosphere, waveguide physics, particle physics, where massive photons are present, sooner or later Čerenkov effect with massive photons will be observed and the theory presented in our paper will be confirmed.

REFERENCES

Anderson, P. W. (1963). Plasmons, gauge invariance, and mass. *Physical Review* **130**(1), 439.

Čerenkov, P. A. (1934). The visible radiation of pure liquids caused by γ-rays. *Comptes Rendus Hebdomaclaires des Scances de l'Academic des Sciences* (USSR) **2**, 451.

Commins, E. D. and Bucksbaum, P. H. (1983). *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge.

de Wit, B. and Smith, J. (1986). *Field Theory in Particle Physics, Vol. I*, Elsevier, Amsterdam.

Dittrich, W. (1978). Source methods in quantum field theory. *Fortschritte der Physik* **26**, 289.

Feldman, G. and Mathews, P. T. (1963). Massive electrodynamics. *Physical Review* **130**, 1633.

Goldhaber, A. S. and Nieto, M. N. (1971). Terrestrial and extraterrestrial limits on the photon mass. *Reviews of Modern Physics* **43**, 277.

Gradshteyn, I. S. and Ryzhik, I. M. (1965). *Table of Integrals, Series and Products*, Academic Press, New York.

Kirzhnitz, A. D. and Linde, A. D. (1972). Macroscopic consequences of the Weinberg model. *Physics Letters* **42B**, 471.

Minkowski, P. and Seiler, R. (1971). Massive vector meson in external fields. *Physical Review D: Particles and Fields* **4**, 359.

Pardy, M. (1983a). Particle production by the Čerenkov mechanism. *Physics Letters* **94A**, 30.
Pardy, M. (1983b). Particle production by accelerated charges moving in condensed matter. *Journal of Physics G: Nuclear Physics* **9**, 853.
Pardy, M. (1989). Finite-temperature Čerenkov radiation. *Physics Letters A* **134**(6), 357.
Pardy, M. (1994a). The Čerenkov effect with radiative corrections. *Physics Letters B* **325**, 517.
Pardy, M. (1994b). The graitational Čerenkov radiation with radiative corrections. *Physics Letters B* **336**, 362.
Pardy, M. (1995). The finite-temperature gravitational Čerenkov radiation. *International Journal of Theoretical Physics* **34**(6), 951.
Ryder, L. H. (1985). *Quantum Field Theory*, Cambridge University Press, Cambridge.
Schwinger, J. (1970). *Particles, Sources and Fields, Vol. I,* Addison-Wesley, Reading, MA.
Schwinger, J., Tsai, W. Y., and Erber, T. (1976). Classical and quantum theory of synergic Synchrotron-Čerenkov radiation. *Annals of Physics* (*New York*) **96**, 303.
Tamm, I. E. and Frank, I. M. (1937). The coherent radiation of a fast electron in a medium. *Doklady Akademil Nauk SSSR* **14**, 107.
van Nieuwenhuizen, P. (1973). Radiation of Massive Gravitons. *Physical Review D: Particles and Fields* **7**(8), 2300.
Voskresensky, D. N., Kolomeitsev, E. E., and Kämpfer, B. (1998). Contribution of the massive photon decay channel to neutrino cooling of neutron stars. e-print: astro-ph/9802251.
Yildiz, A. (1977). Meissner effect in gauge fields. *Physical Review D: Particles and Fields* **16**(12), 3450.